Universal laws and architecture 3:

Foundations for Sustainable Infrastructure

John Doyle John G Braun Professor Control and Dynamical Systems, EE, BE Caltech

Turing on layering

The 'skin of an onion' analogy is also helpful. In considering the functions of the mind or the brain we find certain operations which we can explain in purely mechanical terms. This we say does not correspond to the real mind: it is a sort of skin which we must strip off if we are to find the real mind. But then in what remains we find a further skin to be stripped off, and so on. Proceeding in this way do we ever come to the 'real' mind, or do we eventually come to the skin which has nothing in it? In the latter case the whole mind is mechanical.

1950, Computing Machinery and Intelligence, Mind

"Universal laws and architectures?"

- Universal "conservation laws" (constraints)
- Universal architectures (constraints that deconstrain)

Fundamentals!

A rant

- Mention recent papers*
- Focus on broader context not in papers
- Lots of case studies for motivation

*try to get you to read them?



- Turing 100th birthday in 2012
- Turing
 - machine (math, CS)
 - test (AI, neuroscience)
 - pattern (biology)
- Arguably greatest*
 - all time math/engineering combination
 - WW2 hero
 - "invented" software

Turing (1912-1954)



*Also world-class runner.

Key papers/results

- Theory (1936): Turing machine (TM), computability, (un)decidability, universal machine (UTM)
- Practical design (early 1940s): code-breaking, including the design of code-breaking machines
- Practical design (late 1940s): general purpose digital computers and software, layered architecture
- Theory (1950): Turing test for machine intelligence
- Theory (1952): Reaction diffusion model of morphogenesis, plus practical use of digital computers to simulate biochemical reactions





- Each theory \approx one dimension
- Tradeoffs *across* dimensions
- Assume architectures a priori
- Progress is encouraging, but...
- Stovepipes are an obstacle...



Turing as "new" starting point?



Essentials:

- 0. Model
- 1. Universal laws
- 2. Universal architecture
- 3. Practical implementation

Turing's 3 step research:

- 0. Virtual (TM) machines
- 1. hard limits, (un)decidability using standard model (TM)
- 2. Universal architecture achieving hard limits (UTM)
- 3. Practical implementation in digital electronics (biology?)

Essentials:

- 0. Model
- 1. Universal laws
- 2. Universal architecture
- 3. Practical implementation



Turing's 3 step research:

0. Virtual (TM) machines

- 1. hard limits, (un)decidability using standard model (TM)
- 2. Universal architecture achieving hard limits (UTM)
- 3. Practical implementation in digital electronics (biology?)



- ...being digital should be of greater interest than that of being electronic. That it is electronic is certainly important because these machines owe their high speed to this... But this is virtually all that there is to be said on that subject.
- That the machine is digital however has more subtle significance. ... One can therefore work to any desired degree of accuracy.

1947 Lecture to LMS



- ... digital ... of greater interest than that of being electronic ...
- ...any desired degree of accuracy...
- This accuracy is not obtained by more careful machining of parts, control of temperature variations, and such means, but by a slight increase in the amount of equipment in the machine.

1947 Lecture to LMS

- Digital more important than electronic...
- Robustness: accuracy and repeatability.
- Achieved more by internal hidden complexity than precise components or environments.



Turing Machine (TM)

- Digital
- Symbolic
- Logical
- Repeatable

• ... quite small errors in the initial conditions can have an overwhelming effect at a later time. The displacement of a single electron by a billionth of a centimetre at one moment might make the difference between a man being killed by an avalanche a year later, or escaping.

1950, Computing Machinery and Intelligence, *Mind*

• ... quite small errors in the initial conditions can have an overwhelming effect at a later time....

 It is an essential property of the mechanical systems which we have called 'discrete state machines' that this phenomenon does not occur.

• Even when we consider the actual physical machines instead of the idealised machines, reasonably accurate knowledge of the state at one moment yields reasonably accurate knowledge any number of steps later.

1950, Computing Machinery and Intelligence, Mind



TM Hardware Turing's 3 step research:

- 0. Virtual (TM) machines
- 1. hard limits, (un)decidability using standard model (TM)
- 2. Universal architecture achieving hard limits (UTM)
- 3. Practical implementation in digital electronics (biology?)







time?

Decidable problem = \exists algorithm that solves it

Most naively posed problems are undecidable.



Turing's 3 step research:

- 0. Virtual (TM) machines
- 1. hard limits, (un)decidability using standard model (TM)
- 2. Universal architecture achieving hard limits (UTM)

3. Practical implementation in digital electronics (biology?)





2. Universal architecture achieving hard limits (UTM)

- Software: A Turing machine (TM) can be data for another Turing machine
- A Universal Turing Machine can run any TM
- A UTM is a virtual machine.



The halting problem

- Given a TM (i.e. a computer program)
- Does it halt (or run forever)?
- Or do more or less anything in particular.
- Undecidable! There does not exist a special TM that can tell if any other TM halts.
- i.e. the program HALT does not exist. ⊗

Thm: TM H=HALT does not exist.

That is, there does not exist a program like this:

$$H(TM, input) \triangleq \begin{cases} 1 \text{ if } TM(input) \text{ halts} \\ 0 \text{ otherwise} \end{cases}$$

Proof is by contradiction. Sorry, don't know any alternative. And Turing is a god.

$$H(TM, input) \triangleq \begin{cases} 1 \text{ if } TM(input) \text{ halts} \\ 0 \text{ otherwise} \end{cases}$$

Thm: No such H exists.

Proof: Suppose it does. Then define 2 more programs:

$$H'(TM, input) \triangleq \begin{cases} 1 \text{ if } H(TM, input) = 0\\ \text{loop forever otherwise} \end{cases}$$

$$H^{*}(TM) \triangleq H'(TM, TM)$$

Run $H^*(H^*) = H'(H^*, H^*)$ = $\begin{cases} \text{halt if } H^*(H^*) \text{ loops forever} \\ \text{loop forever otherwise} \end{cases}$

Contradiction!



Implications

- TMs and UTMs are perfectly repeatable
- But perfectly unpredictable
- Undecidable: Will a TM halt? Is a TM a UTM? Does a TM do X (for almost any X)?
- Easy to make UTMs, but hard to recognize them.
- Is anything decidable? Yes, many questions NOT about TMs.
- Large, thin, nonconvex everywhere...

Issues for engineering

- Turing remarkably relevant for 76 years
- UTMs are \approx implementable
 - Time is most critical resource
 - Space (memory) almost free
- Read/write random access memory hierarchies
- Further gradations of decidable (P/NP/coNP)

 Must crucial: You can fix bugs but it is hard to automate finding/avoiding them Issues for neuroscience

- Brains and UTMs?
 - Time is most critical resource?
 - Space (memory) almost free?
- Read/write random access memory hierarchies?
- Brain >> UTM?

Gallistel and King



Memory and the Computational Brain

Why Cognitive Science Will Transform Neuroscience

WILEY-BLACKWELL

Conjecture

- Memory potential $\approx \infty$
- Examples
 - Insects
 - Scrub jays
 - Autistic Savants

Gallistel and King



Memory and the Computational Brain

Why Cognitive Science Will Transform Neuroscience

WILEY-BLACKWELL

- But why so rare and/or accidental?
- Large memory, computation of limited value?
- Selection favors fast robust action?







Communications

Shannon's brilliant insight

- Don't worry about time or delay!
- Don't compress and code files, worry only about *infinite random ensembles*
- Information theory is most popular and accessible topic in systems engineering
- *Fantastic* for engineering, almost useless for biology (But see Lestas, Vinnicombe, Paulsson)
- (And largely irrelevant to Internet architecture)
- Misled and distracted generations of biologists and neuroscientists
- New generation of information theorists are putting delay back in. (Cheer!)

Shannon





of intermediate alternatives.



Want to emphasize the differences between these two types of layering.












Essentials To Do

- Reyna/Brainerd: Gist, false memory
- Ashby: Automaticity, multiple memory systems,...
- Cosmides/Tooby: Risk, uncertainty, cooperation, evolution,...



Speed and flexibility are crucial to implementing robust controllers.



Speed and flexibility are crucial to implementing robust controllers.

Beyond black boxes:

Putting brain physiology back in the picture

Essentials (following Turing)

- 0. Model
- 1. Universal laws
- 2. Universal architecture
- 3. Practical implementation











Cyberphysical



Turing as "new" starting point?

Essentials:

- 0. Model
- 1. Universal laws
- 2. Universal architecture
- 3. Practical implementation



Maybe start from here with Turing's 3 step research:

- 1. hard limits, (un)decidability using standard model (TM)
- 2. Universal architecture achieving hard limits (UTM)
- 3. Practical implementation in digital electronics (biology?)





- Each theory \approx one dimension
- Tradeoffs *across* dimensions
- Assume architectures a priori
- Progress is encouraging, but...
- Stovepipes are an obstacle...





laws and architectures?



Viruses' Life History: Towards a Mechanistic Basis of a Trade-Off between Survival and Reproduction among Phages

Marianne De Paepe, François Taddei^{*}

Laboratoire de Genetique Moleculaire, Evolutive et Medicale, University of Paris 5, INSERM, Paris, France

Marianne De Paepe, François Taddei^{*}

Laboratoire de Genetique Moleculaire, Evolutive et Medicale, University of Paris 5, INSERM, Paris, France

July 2006 | Volume 4 | Issue 7 | e193



Phage

1μm

Bacteria

Phage lifecycle







Traits and Phage Particle Characteristics







Name	Type of Phage		Measured Life Cycle Characteristics						Published Structural Properties			Calculated Ratio	
	Family	Life Cycle	Decay Rate (d)		Latency Period (min)	Multiplication Rate ^a (h ⁻¹)	Adsorption Rate (min ⁻¹)	E _x b (kJ/mol)	Genome Size (kb)	Ext. Diameter ^c (nm)	Capsid MW ^d (kDa)	Surfacic Mass ^e (kDa/nm²)	Ppack
λ	Siphoviridae	т	0.072	115	42	162	4.5×10^{-10}	142	49 [37]	63 [24]	22,500 [38]	22.7	0.572
M13	Inoviridae	Chronic				413	9.0×10^{-11}	125	6 [37]	6.5×90 [37]	15,700 [39]	8.7	
MS2	Leviviridae	L	0.250	400	40	669	6.5×10^{-10}	99	4 [37]	27 [40]	2,500 [41]	13.7	
Mu	Myoviridae	Т	0.290	200	60	200	φ	111	43 [37]	54 [42]	15,000 [43]	20.6	0.845
P1	Myoviridae	Т	0.077	400	60	149	2.2×10^{-10}	119	100 [37]	85 [44]			0.435
P2	Myoviridae	Т	0.041	160	48	88	5.5×10^{-11}	123	34 [37]	60 [45]	20,400 [46]	22.7	0.468
P4	Myoviridae	Т	0.045	300	60	101	2.2×10^{-10}	105	12 [37]	45 [46]	12,400 [46]	24.5	0.429
φ80	Siphoviridae	Т	0.120	600	55	776	3.8×10^{-10}	114	45	61		24.3	0.585
φX174	Microviridae	L	0.200	180	15	697	2.9×10^{-9}	136	5 [37]	32 [47]	4,700 [48]	18.4	
PRD1	Tectiviridae	L	0.037	50	48	50	4.6×10^{-10}	171	15 [49]	65 [49]	33,000 [49]	35.5	0.421
T2	Myoviridae	L	0.068	135	23	335	4.0×10^{-10}		170 [37]	85x110 [50]		19.9	0.451
T3	Podoviridae	L	0.102	200	17	700	1.6×10^{-9}	105	38 [51]	60 [52]		18.1	0.525
T4	Myoviridae	L	0.068	150	23	400	5.0×10^{-10}	96	170 [37]	85x110 [50]	65,600 [50]	26.9	0.421
TS	Siphoviridae	L	0.120	290	44	399	2.0×10^{-10}	115	122 [53]	65 [53]	27,500 [53]	13.7	0.439
T7	Podoviridae	L	0.187	260	13	1,131	3.0×10^{-9}	100	40 [37]	60 [52]	16,300 [54]	19.4	0.615
R17	Leviviridae	L	0.520	3,570	53	4,288	3.7×10^{-9}	99	4 [37]	27 [55]	2,600 [41]	14.7	

Mortality rate, burst size, latency period, and adsorption rate were measured as described in Material and Methods. Each value is the mean of at least three independent experiments. Genome size, diameter, and molecular weight were collected from published results. The internal volume used to calculate ρ_{pack} has either been collected in structural studies of phage capsids or calculated by subtracting the thickness of the shell from the external diameter. Empty cells in the table correspond to data that were either not available or not measured. ^aMean of the ratio obtained by dividing the burst size by the latency period, calculated for each experiment.

^bE₀: energy of activation of the reaction leading to inactivation of virions, obtained from the Arrhenius equation linking mortality rate and temperature between 30 °C and 45 °C. The energy of activation represents the energy the system has to overcome so that the reaction occurs.

^cExt. diameter: external diameter of the capsid.

^dMolecular weight of the proteins constituting the capsid.

*Capsid molecular weight divided by the surface of the capsid; this ratio represents the thickness of the shell.

Volume occupied by the genome divided by the internal volume of the capsid.

T: Temperate phage, L: Virulent Phage, Chronic: creates a chronic infection

DOI: 10.1371/journal.pbio.0040193.t001

$1 \mu m$







1μm

Bacterium (Chlamydia)

Bacterium (Staph. aureus)



Pox virus







Herpes

Influenza

Polio

Bacterium (Staphyllococcus aureus)









IRES

7500 nucleotides ≈ 15kbits **slow**



Glycolytic Oscillations and Limits on Robust Efficiency

Fiona A. Chandra,¹* Gentian Buzi,² John C. Doyle²

Both engineering and evolution are constrained by trade-offs between efficiency and robustness, but theory that formalizes this fact is limited. For a simple two-state model of glycolysis, we explicitly derive analytic equations for hard trade-offs between robustness and efficiency with oscillations as an inevitable side effect. The model describes how the trade-offs arise from individual parameters, including the interplay of feedback control with autocatalysis of network products necessary to power and catalyze intermediate reactions. We then use control theory to prove that the essential features of these hard trade-off "laws" are universal and fundamental, in that they depend minimally on the details of this system and generalize to the robust efficiency of any autocatalytic network. The theory also suggests worst-case conditions that are consistent with initial experiments.

Chandra, Buzi, and Doyle

Most important paper so far.

UG biochem, math, control theory

the cen's use of ATF. In giveorysis, two ATP molecules are consumed upstream and four are produced downstream, which normalizes to q = 1(each y molecule produces two downstream) with kinetic exponent a = 1. To highlight essential trade-offs with the simplest possible analysis, we normalize the concentration such that the unperturbed ($\delta = 0$) steady states are $\overline{y} = 1$ and $\overline{x} = 1/k$ [the system can have one additional steady state, which is unstable when (1, 1/k) is stable]. [See the supporting online material (SOM) part I]. The basal rate of the PFK reaction and the consumption rate have been normalized to 1 (the 2 in the numerator and feedback coefficients of the reactions come from these normalizations). Our results hold for more general systems as discussed below and in SOM, but the analysis



www.sciencemag.org SCIENCE VOL 333 8 JULY 2011





Figure S4. Simulation of two state model (S7.1) qualitatively recapitulates experimental observation from CSTR studies [5] and [12]. As the flow of material in/out of the system is increased, the system enters a limit cycle and then stabilizes again. For this simulation, we take q=a=Vm=1, k=0.2, g=1, u=0.01, h=2.5.



Why?

Levels of explanation:

- 1. Possible
- 2. Plausible
- 3. Actual



- 4. Mechanistic
- 5. Necessary

Engineering Medicine

Glycolytic "circuit" and oscillations

- Most studied, persistent mystery in cell dynamics
- End of an old story (why oscillations)
 - side effect of hard robustness/efficiency tradeoffs
 - no purpose per se
 - just needed a theorem
- Beginning of a new one
 - robustness/efficiency tradeoffs
 - complexity and architecture
 - need more theorems and applications





Enzyme amount

Savageaumics







energy Rest of cell

Yeast anaerobic glycolysis




control feedback



Tight control creates "weak linkage" between power supply and demand



Tight control creates "weak linkage" between power supply and demand





enzymes catalyze reactions





Efficient = low enzyme amount



Standard story: Autocatalytic plus control feedback necessary and sufficient for oscillations

Proof: Dynamical systems model, simulation, bifurcation analysis







There may be other resources needed that aren't recycled

which we'll ignore for now



Some processes don't require autocatalysis









Maintain product?





Cut back product



Cut back product more



Over-react and oscillate





Control feedback



Autocatalytic feedback makes control harder











Standard story: Autocatalytic plus control feedback necessary and sufficient for oscillations

Proof: Dynamical systems model, simulation, bifurcation analysis







Fluorescence histogram (fluorescence vs. cell count) of GFP-tagged Glyceraldehyde-3-phosphate dehydrogenase (TDH3). Cells grown in ethanol have lower mean and median and higher variability.





See Lestas, Vinnicombe, Paulsson, Nature





- Transcription is highly variable
- Even if you allow ∞ delay!
- Information theory applies

See Lestas, Vinnicombe, Paulsson, Nature







a=1 *sufficient* for oscillations (and is *actual*) g=1 necessary for robust efficiency (and is *actual*)

a=1

 10^{0}

 $\geq \ln \left| \frac{z+p}{z-p} \right|$

Ŀ

 10^{1}

a=g=1

*a=g=*0

z+p

 $\overline{z-p}$


a=1 sufficient for oscillations (and is actual) g=1 necessary for robust efficiency (and is actual)

autocatalytic



What (some) reviewers say

- "...to establish universality for all biological and physiological systems is **simply wrong**. It cannot be done..."
- "... a mathematical scheme without any real connections to biological or medical..."
- "If such oscillations are indeed optimal, why are they not universally present?"
- "...universality is well justified in physics... for biological and physiological systems ...a dream that will never be realized, due to the vast diversity in such systems."
- "...does not seem to understand or appreciate the vast diversity of biological and physiological systems..."
- "...a high degree of abstraction, which ...make[s] the model useless ..."





Enzyme amount

Savageaumics

Architecture

Good architectures allow for effective tradeoffs



wasteful





How general is this picture?



Supplementary materials has a demo.

Architecture, constraints, and behavior

John C. Doyle^{a,1} and Marie Csete^{b,1}

^aControl and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125; and ^bDepartment of Anesthesiology, University of California, San Diego, CA 92103

Edited by Donald W. Pfaff, The Rockefeller University, New York, NY, and approved June 10, 2011 (received for review-March 3, 201

This paper aims to bridge progress in neuroscience involving sophisticated guantitative analysis of behavior, including the use of robust control, with other relevant conceptual and theoretical frameworks from systems engineering, systems biology, and mathematics. Familiar and accessible case studies are used to illustrate concepts of robustness, organization, and architecture (modularity and protocols) that are central to understanding complex networks. These essential organizational features are hidden during normal function of a system but are fundamental for understanding the nature, design, and function of complex biologic and technologic systems.

evolved for sensorimotor control and retain m architecture, then the apparent distinctions be cognitive, and motor processes may be anoth (9), reinforcing the claim that robust con feedback (7, 11) rather than more convent processing might be more useful in interpretic data (9). This view also seems broadly arguments from grounded cognition that bodily states, and situated action underly not but cognition in general (12), including language (13). Furthermore the myriad constraints involved in the evolution of circuit

h of that evolved

ween perceptual.

form of illusion

and adaptive

nal serial signal

ner

OT COL

Doyle and Csete, Proc Nat Acad Sci USA, online JULY 25 2011









Robust =agile and balancing





Efficient=length of pendulum (artificial)



$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \frac{1}{D(s)} \begin{bmatrix} ls^2 \pm g \\ -s^2 \end{bmatrix} u \qquad D(s) = s^2 (Mls^2 \pm (M+m)g)$$
$$y = x + \alpha l\theta = \frac{(1-\alpha)ls^2 \pm g}{D(s)}$$
$$p = \sqrt{\frac{g}{l}} \sqrt{1+r} \quad r = \frac{m}{M} \quad z = \sqrt{\frac{g}{l}} \sqrt{\frac{1}{(1-\alpha)}}$$
transform+algebra

$$(M + m)\ddot{x} + ml\ddot{\theta} = u$$
$$\ddot{x} + l\ddot{\theta} \pm g\theta = 0$$
$$y = x + \alpha l\theta$$



 $\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\left(j\omega\right)\right|d\omega\geq0$



Easy, even with eyes closed No matter what the length

Proof: Standard UG control theory: Easy calculus, easier contour integral, easiest Poisson Integral formula

 ∞ $\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\left(j\omega\right)\right|d\omega\geq0$

 \approx simple metabolism without autocatalysis



Need vision



Harder if delayed or short



Also harder if low

 $r = \frac{m}{M}$



Gratuitous fragility versus fragile robustness

$$\int_{0}^{\infty} \ln \left| S(j\omega) \right| d\omega \ge 0$$

- \gg \Rightarrow Gratuitous fragility
- \Rightarrow Fragile robustness

Theorem is independent of control implementation.

Simple metabolism without autocatalysis



Up is hard for shorter lengths (RHP pole)

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\left(j\omega\right)\right|d\omega\geq\left|p\right|$$



$$p = \sqrt{\frac{g}{l}(1+r)}$$
 $r = \frac{m}{M}$





 $p \text{ small} \Leftrightarrow l \text{ large}$

Down easy, even with

- eyes closed
- all lengths

 $\int \ln \left| S(j\omega) \right| d\omega \ge \left| p \right|$ π

This is a cartoon, but can be made precise.



Eyes moved down is harder (RHP zero) Similar to delay



M

 $r = \frac{m}{M}$





M

Compare

$$p = \sqrt{\frac{g}{l(1-\varepsilon)}} \sqrt{1+r} = p_0 \sqrt{\frac{1}{(1-\varepsilon)}} \approx p_0 \left(1 + \frac{\varepsilon}{2}\right)$$

Move eyes

$$p = \sqrt{\frac{g}{l}}\sqrt{1+r} \quad r = \frac{m}{M} \quad z = \sqrt{\frac{g}{l}}\sqrt{\frac{1}{\varepsilon}}$$

$$p = z \Longrightarrow 1+r = \frac{1}{\varepsilon} \Longrightarrow \varepsilon = \frac{1}{1+r}$$

$$p\left(1+\frac{1}{3}\frac{p^2}{z^2}\right) = \sqrt{\frac{g}{l}}\sqrt{1+r}\left(1+\frac{1}{3}\varepsilon\right) = p\left(1+\frac{\varepsilon}{3}\right)$$

$$= p\left(1+\frac{1-\alpha}{3}\right)$$





This is a cartoon, but can be made precise.

Next (and last) time

- Universal laws in more depth
- Universal architectures revisited/compared
 - Computers and networks
 - Cells
 - Brains and minds
- Architecture & laws at the extremes
 - evolution
 - eusociality







Translation: Amino acids polymerized into proteins



Ribosomes are made of proteins and RNA



Ribosomes are made of proteins and rRNA






Evolution and architecture

Nothing in biology makes sense except in the light of evolution

Theodosius Dobzhansky

(see also de Chardin)

Nothing in evolution makes sense except in the light of biology

?????

Standard theory: natural selection + genetic drift + mutation + gene flow

Greatly abridged cartoon here



Shapiro explains well what this is and why it's incomplete (but Koonin is more mainstream)

Standard theory: selection + drift + mutation + gene flow



Standard theory: selection + drift + mutation + gene flow







All complexity is emergent from random ensembles with minimal tuning.

No new laws.

No architecture.

The battleground





No gap. Just physics. Huge gap. Need supernatural



Genes?

What they agree on

No new laws. No architecture. No biology.





Huge gap.







Putting biology back into evolution





The heresies

- Many mechanisms for "horizontal" gene transfer
- Many mechanisms to create large, functional mutations
- At highly variable rate, can be huge, global
- Selection alone is a very limited filtering mechanism
- Mutations *can* be "targeted" within the genomes
- *Can* coordinate DNA change w/ useful adaptive needs
- Viruses *can* induce DNA change giving heritable resistance
- Still myopic about future, still produces the grotesque

THE SOCIAL CONQUEST OF EARTH

Surprising heresies from "conservatives"



E D WA R D O. W I L S O N

WINNER of the PULITZER PRIZE



The Logic of Chance

The Nature and Origin of Biological Evolution

EUGENE V. KOONIN



selection + drift + mutation + gene flow + facilitated variation



large functional changes in genomes

natural selection + genetic drift + mutation + gene flow + facilitated variation





natural selection + genetic drift + mutation + gene flow + facilitated variation





Reading?

- See refs in 2011 PNAS paper but also...
- Turing: Gallistel (+ Wolpert on control/bayes)
- Brain/Mind: Gazzaniga, Kahneman + Reyna/Brainerd, Ashby, Cosmides/Tooby,...
- Evolution: Gerhart & Kirschner, Shapiro, Lane, Koonin, Caporale (+ fire + running)
- Apes: De Waal (Bonobos), Sapolsky (Baboons)
- Eusociality: Wilson
- Juarrero